

**INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH  
TECHNOLOGY****STUDY OF ROUNDNESS ERRORS MEASUREMENT ON CMM USING A  
SEPARATION METHOD****Loubna Laaouina<sup>1</sup>, Abdelhak Nafi<sup>1</sup>, Ahmed Mouchtachi<sup>2</sup>**<sup>1</sup>Laboratory of Advanced Mechanics Research and Industrial Application ,Superior National School of Arts and Crafts (ENSAM), Moulay Ismail University, Meknes, Morocco<sup>2</sup>Laboratory of renewable energy and sustainable development, Superior National School of Arts and Crafts (ENSAM ), Hassan II University, Casablanca, Morocco

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**ABSTRACT**

The accuracy of roundness measurement on CMM is in majority affected by the probe errors and geometric errors related to the machine axes. The separation technique is an approved method for decoupling the probe errors from other source of errors. This paper presents a novel application and a study of the separation method, to measure the roundness errors separately to the probe errors and machine errors. The proposed approach has been tested and validated using several virtual machines.

**KEYWORDS:** Coordinate measuring machine, machine errors, metrology, pre-travel, probe errors, roundness measurement.

**INTRODUCTION**

Development of efficient techniques to verify the performance of coordinate measuring machines has been the subject of many researches...[1-2-3]. Indeed, several sources of errors affect the accuracy of CMMs such as probe errors and machine errors that include dynamic and kinematics errors. The measurement of machine errors depends also on the measurement volume. When measuring an artifact in a small measurement volume of the CMM, it's known that, the effect of the machine axes are not dominant so that the observed errors are for the majority caused by the probe [4-5].

The systematic probe error is the pre-travel, it is the difference between the machine position when contact occurs and the triggering position. There are several methods to determine the pre-travel value based on direct measurements using specific setup or by using an analytical model [6-7-8]. In case of the specific setup, the stylus tip is moved through a low force system; the initial contact just before the stylus tip moves is detected by a high sensitivity mechanical system which then continues moving the stylus tip until triggering occurs.

The roundness measurement, on a CMM, is affected by the machine errors and the probe errors. Therefore, each probed point on the part is associated to a combination of three types of errors. The development of a method for the separation of these three errors could be a good contribution in the field of metrology. The measurement of roundness errors separately to a measuring system has been the subject of several research and variety of methods have been proposed [9-10-11].

The multistep method was implemented for decoupling probe errors and the rest of the machine errors[12-13] ,it's useful to evaluate in a production environment and determine if the probe or the rest of the machine are causing measurement errors .

For further benefit from the multistep method on a CMM. This paper presents a development of this method for decoupling probe errors, machine errors and also roundness errors of the test ring. The proposed approach has been tested and validated using several virtual machines.

### ROUNDNESS MEASUREMENT ON CMM USING THE SEPARATION METHOD

The method is based on probing  $n$  points on a circle of the inner diameter of the test ring. This is for  $n$  configurations of the test ring and  $n$  configurations of the probing system; each configuration is defined by a rotation around a vertical axis by an increment of  $360^\circ/n$ . For one configuration of the test ring and one configuration of the probing system, there are  $n$  measured points on the circle, thus combining  $n$  probe errors,  $n$  roundness errors of the test ring and  $n$  machine errors.

For the first configuration of the test ring, a circle of the inner diameter is measured following  $n$  configurations of the probing system. For the second configuration of the test ring, a same circle of the inner diameter is re-measured following the  $n$  configurations of the probing system. This process is repeated until all  $n$  configurations of the test ring are done. For one configuration to another, the same points on the test ring are probed. We have a permutation of probe errors and roundness errors on all measured points. However, the machine errors remain the same. Figure 1 illustrates the test ring, probing system and rest of the machine.

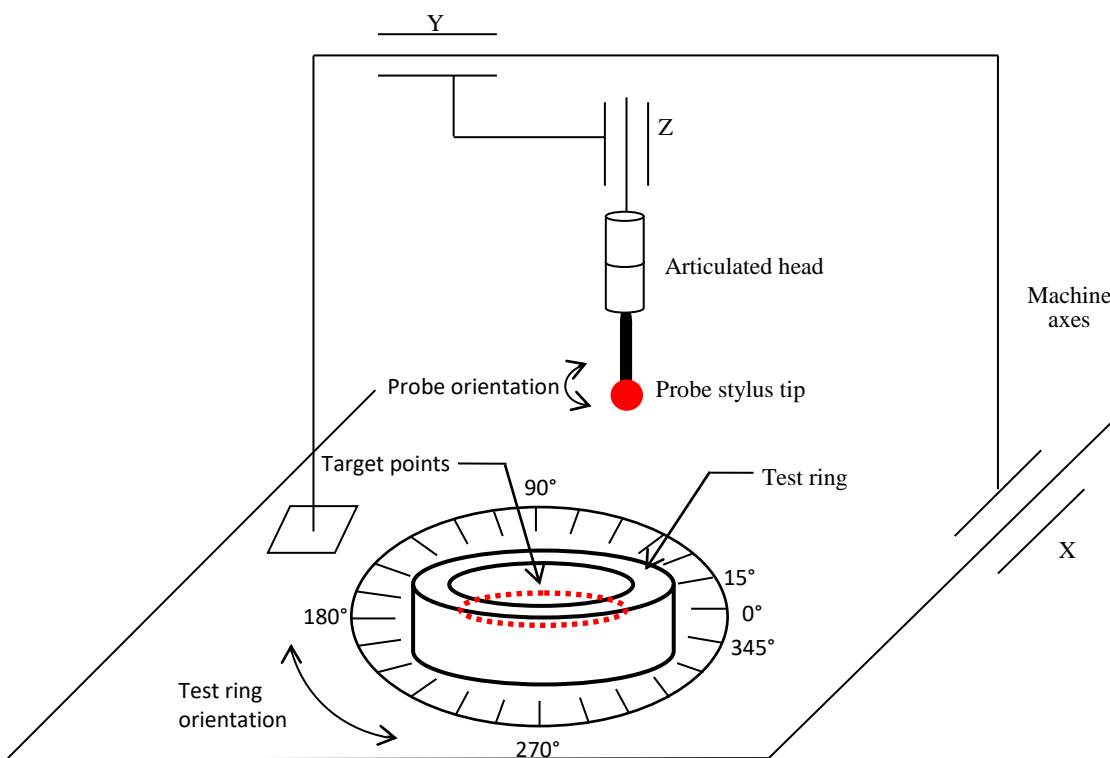


Figure 1: Test ring, probing system and rest of the machine

For each configuration of the test ring, there are  $n$  configurations of probing system. The radial residual  $e$  between the measured radius of the test ring and the theoretical radius at each probed point is explained by three errors that are: probe error  $\delta_p$ , roundness errors of the test ring  $\delta_{fr}$  and error associated to the machine axes  $\delta_m$ . For each measured point, the following equation applies:

$$e_{k,t,r} = -\delta_{p,i} + \delta_{fr,j} + \delta_{m,k} \quad (1)$$

Where  $t$  indicates the configuration of the probing system,  $r$  indicates the configuration of the test ring,  $i$  indicates the probe errors,  $j$  indicates the roundness error of the test ring,  $k$  indicates the machine approach direction.

For configuration  $t$  of the probing system and configuration  $r$  of the test ring, we have a subsystem of  $n$  equations corresponding to  $n$  machine approach direction.

$$[e_k]_{(t,r)} = \begin{bmatrix} \mathbf{0}_{(t-1) \times (n-(t-1))} & \vdots & -\mathbf{I}_{(t-1) \times (t-1)} & \vdots & \mathbf{0}_{(r-1) \times (n-(r-1))} & \vdots & \mathbf{I}_{(r-1) \times (r-1)} & \vdots \\ \dots & \vdots & \dots & \vdots & \dots & \vdots & \dots & \vdots \\ -\mathbf{I}_{(n-(t-1)) \times (n-(t-1))} & \vdots & \mathbf{0}_{(n-(t-1)) \times (t-1)} & \vdots & \mathbf{I}_{(n-(r-1)) \times (n-(r-1))} & \vdots & \mathbf{0}_{(n-(r-1)) \times (r-1)} & \vdots \end{bmatrix} [\delta] \quad (2)$$

$[e_k]_{(t,r)}$  is a column vectors (n by 1) which contains the n radials residual ( $k=1,2,3,\dots,n$ ).

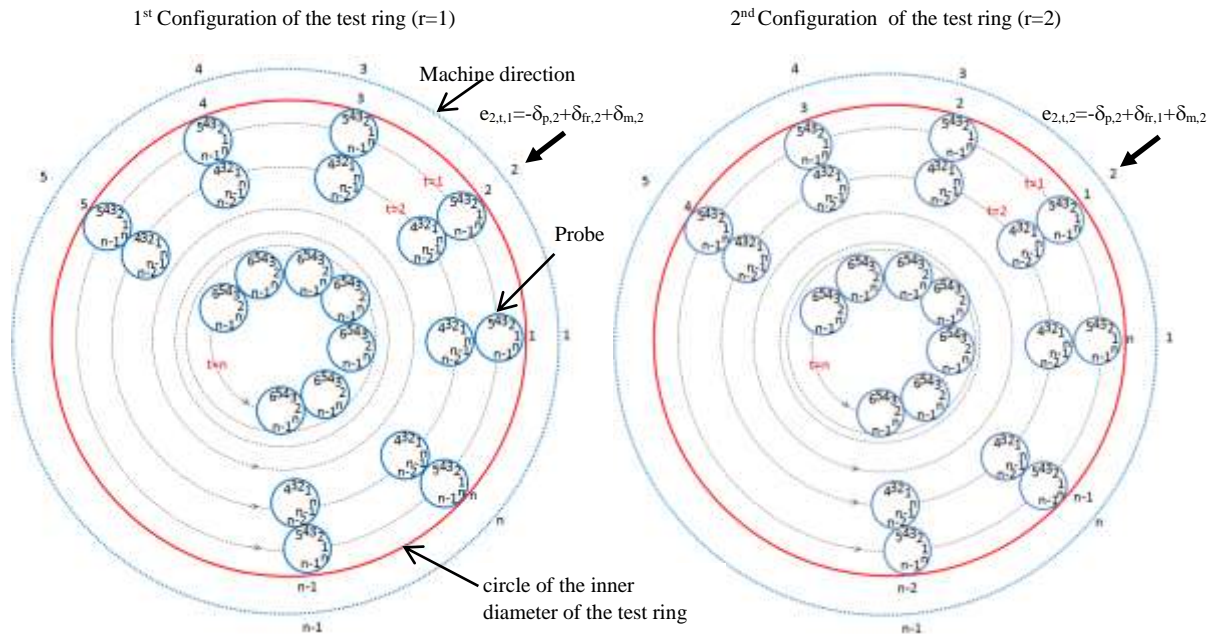
$[\delta]$  is a column vectors (3n by 1) which contains the 3n errors sources (n probe errors, n roundness errors and n machine errors)

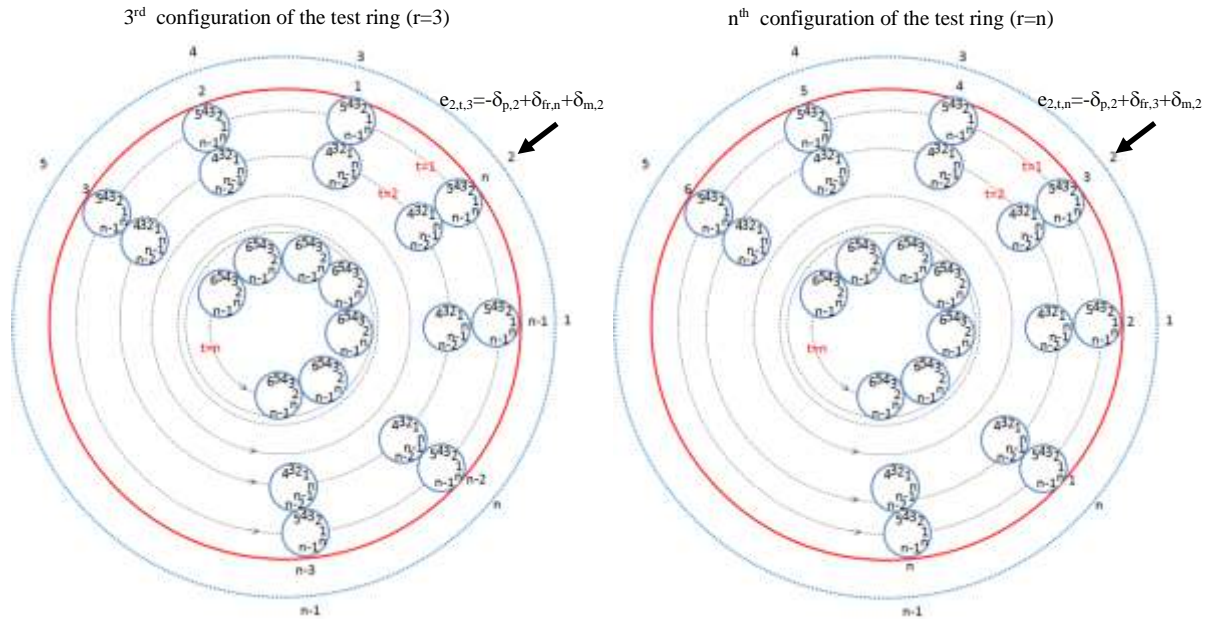
The identification matrix for this subsystem is (n by 3n) it's constructed by some submatrices (identity matrix and zero matrix), for example

$\mathbf{I}_{(t-1) \times (t-1)}$  is an (t-1) by (t-1) identity matrix

$\mathbf{0}_{(t-1) \times (n-(t-1))}$  is an (t-1) by (n-(t-1)) zero matrix

Figure 2 illustrates the measurement sequence of the test ring with some probe configurations.





**Figure 2: The measurement sequence of the test ring for different test ring configurations and probe configurations**

For the first configuration of the test ring  $r=1$ , we proceed to create a subsystem for each configuration of the probing system  $t=1, 2, 3, \dots, n$ .  
 For the first configuration of probing system  $t=1$  a subsystem is:

$$[e_k]_{(1,1)} = [-I_{n \times n} \quad \vdots \quad I_{n \times n} \quad \vdots \quad I_{n \times n}] [\delta] \tag{3}$$

For the second configuration of probing system  $t=2$  a subsystem is:

$$[e_k]_{(2,1)} = \begin{bmatrix} 0_{1 \times (n-1)} & \vdots & -1 & \vdots & \vdots & \vdots \\ \dots & \vdots & \dots & \vdots & I_{n \times n} & I_{n \times n} \\ -I_{(n-1) \times (n-1)} & \vdots & 0_{(n-1) \times 1} & \vdots & \vdots & \vdots \end{bmatrix} [\delta] \tag{4}$$

For the  $n^{\text{th}}$  configuration of probing system  $t=n$  a subsystem is:

$$[e_k]_{(n,1)} = \begin{bmatrix} 0_{(n-1) \times 1} & \vdots & -I_{(n-1) \times (n-1)} & \vdots & \vdots & \vdots \\ \dots & \vdots & \dots & \vdots & I_{n \times n} & I_{n \times n} \\ -1 & \vdots & 0_{1 \times (n-1)} & \vdots & \vdots & \vdots \end{bmatrix} [\delta] \tag{5}$$

We have  $n$  subsystems corresponding to the first configuration of the test ring  
 For the second configuration of the test ring, we have  $n$  other subsystems and so on for all configurations of the test ring. The total number of subsystem for all configurations is  $n^2$   
 By considering  $n \times n$  configurations ( $n$  configurations of the test ring and  $n$  configurations of the probe system), a linear system is composed from the concatenation of all subsystems.

$$E = M \delta_t \tag{6}$$

Where E: column vectors ( $n^3 \times 1$ ) containing all radial residuals measured for all configurations;  
 M: identification matrix ( $n^3 \times 3n$ );  
 $\delta$ : column vectors ( $3n \times 1$ ) containing n probe errors, n roundness errors of the test ring and n machine errors.

**Table 1: Example of subsystems for n=3**

	r=1	r=2	r=3
t=1	$\begin{bmatrix} e_{1,1,1} \\ e_{2,1,1} \\ e_{3,1,1} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} [\delta]$	$\begin{bmatrix} e_{1,1,2} \\ e_{2,1,2} \\ e_{3,1,2} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} [\delta]$	$\begin{bmatrix} e_{1,1,3} \\ e_{2,1,3} \\ e_{3,1,3} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} [\delta]$
t=2	$\begin{bmatrix} e_{1,2,1} \\ e_{2,2,1} \\ e_{3,2,1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} [\delta]$	$\begin{bmatrix} e_{1,2,2} \\ e_{2,2,2} \\ e_{3,2,2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} [\delta]$	$\begin{bmatrix} e_{1,2,3} \\ e_{2,2,3} \\ e_{3,2,3} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} [\delta]$
t=3	$\begin{bmatrix} e_{1,3,1} \\ e_{2,3,1} \\ e_{3,3,1} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} [\delta]$	$\begin{bmatrix} e_{1,3,2} \\ e_{2,3,2} \\ e_{3,3,2} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} [\delta]$	$\begin{bmatrix} e_{1,3,3} \\ e_{2,3,3} \\ e_{3,3,3} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} [\delta]$

For all configurations,  $[\delta] = [\delta_{p,1} \ \delta_{p,2} \ \delta_{p,3} \ \delta_{fr,1} \ \delta_{fr,2} \ \delta_{fr,3} \ \delta_{m,1} \ \delta_{m,2} \ \delta_{m,3}]^T$  is the same, it contains nine errors sources, three probe errors, three out of roundness errors and three machine errors,  $[\delta]$  explains all 27 measured errors  $e_{k,t,r}$ .

**ANALYSIS OF THE IDENTIFICATION SYSTEM**

In this study, we have created system of equations with n=24. The identification of probe errors, out of roundness errors of the test ring and machine errors is mainly depends on the analysis of the matrix M. The rank of this matrix is 70, it's deficient by two, since there are 72 columns in the matrix corresponding to 72 unknowns. The condition number is infinite .To solve the system and have the absolute probe errors, out of roundness of the test ring and absolute machine errors, two more equations would be needed. To deal with this issue, we propose to set sequentially two errors by removing the associated columns from the system, while controlling the rank and the condition number of the new reduced matrix.

Let us consider:

$P_i$  is the  $i^{th}$  column associated to probe error  $\delta_{p,i}$ ;

$S_i$  is the  $i^{th}$  column associated to out of roundness of the test ring  $\delta_{fr,i}$ ;

$M_i$  is the  $i^{th}$  column associated to machine error  $\delta_{m,i}$ .

Firstly, we start by removing the first column  $P_1$ , then we remove the second column  $P_2$  or  $P_3 \dots$  or  $P_{24}$  or  $S_1$  or  $S_2 \dots$  or  $S_{24}$  or  $M_1$  or  $M_2 \dots$  or  $M_{24}$ .

Example: we remove the column  $P_1$ , if the second column removing is  $P_2$ , in this case we have the configuration  $P_1P_2$ . If the second column removing is  $S_2$ , in this case we have the configuration  $P_1S_2$ .

The all configurations for this case are:  $P_1P_2, P_1P_3, \dots, P_1P_{24}, P_1S_1, P_1S_2, \dots, P_1S_{24}, P_1M_1, P_1M_2, \dots, P_1M_{24}$

Secondly, we start by removing the first column  $S_1$ , then we remove the second column  $S_2$  or  $S_3 \dots$  or  $S_{24}$  or  $M_1$  or  $M_2 \dots$  or  $M_{24}$ .

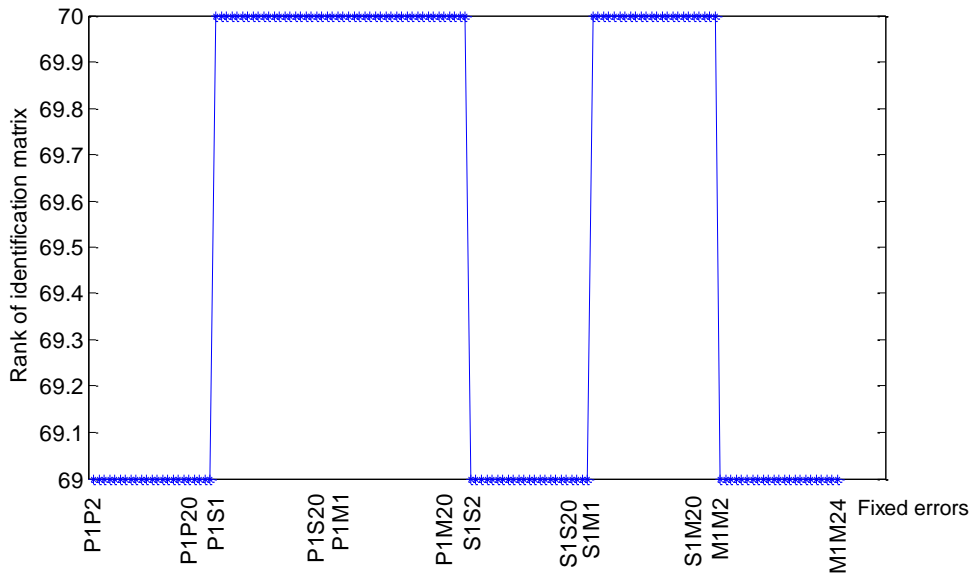
The all configurations for this case are:  $S_1S_2, \dots, S_1S_{24}, S_1M_1, S_1M_2, \dots, S_1M_{24}$

Thirdly, we start by removing the first column  $M_1$ , then we remove the second column  $M_2$  or  $M_3 \dots$  or  $M_{24}$

The all configurations for this case are:  $M_1M_2, M_1M_3, \dots, M_1M_{24}$ .

The figure 3 shows the rank and the condition number for all studied configurations.

a)



b)

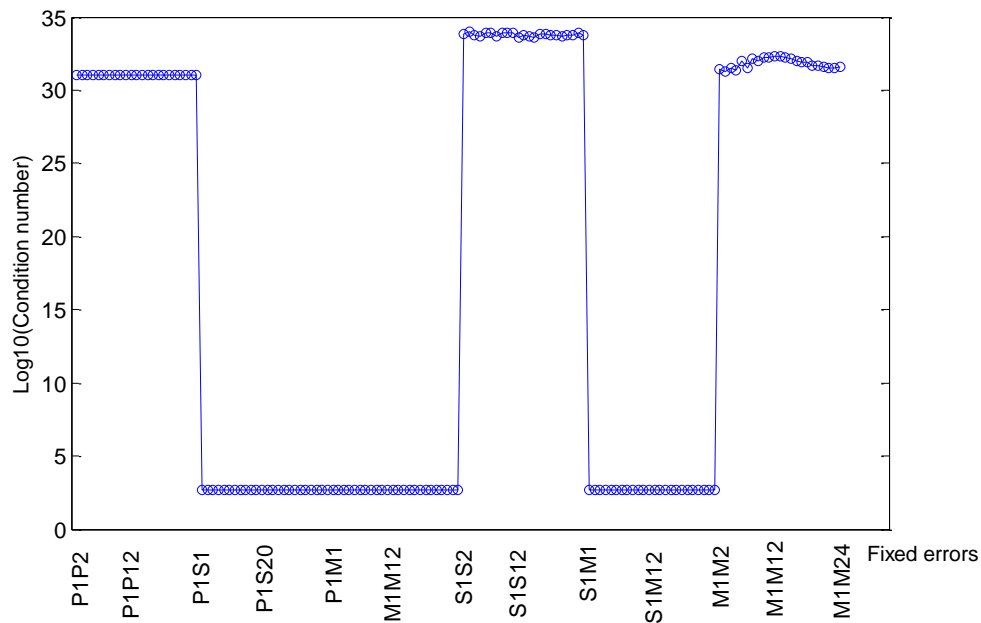


Figure 3: (a) Rank and (b) condition number of reduced identification matrix

By removing, from the matrix  $M$ , two columns corresponding to two errors which have the same origin (configurations  $P_1P_k$ ,  $S_1S_k$  or  $M_1M_k$ , where  $k=1,2,3,\dots,24$ ), the rank is 69 and the condition number of the reduced matrix is very high of order  $10^{13}$ . However if we remove, from the matrix  $M$ , two columns corresponding to two errors which have not the same origin (configurations  $P_1S_k$ ,  $P_1M_k$  or  $S_1M_k$ , where  $k=1,2,3,\dots,24$ ), the rank is 70 and the condition number is about 14. In order to estimate the probe errors, the roundness error of the test ring and the machine errors, we have to set at least two errors (one probe error and one machine error) or (one probe error and one value of roundness error) or (one machine error and one value of

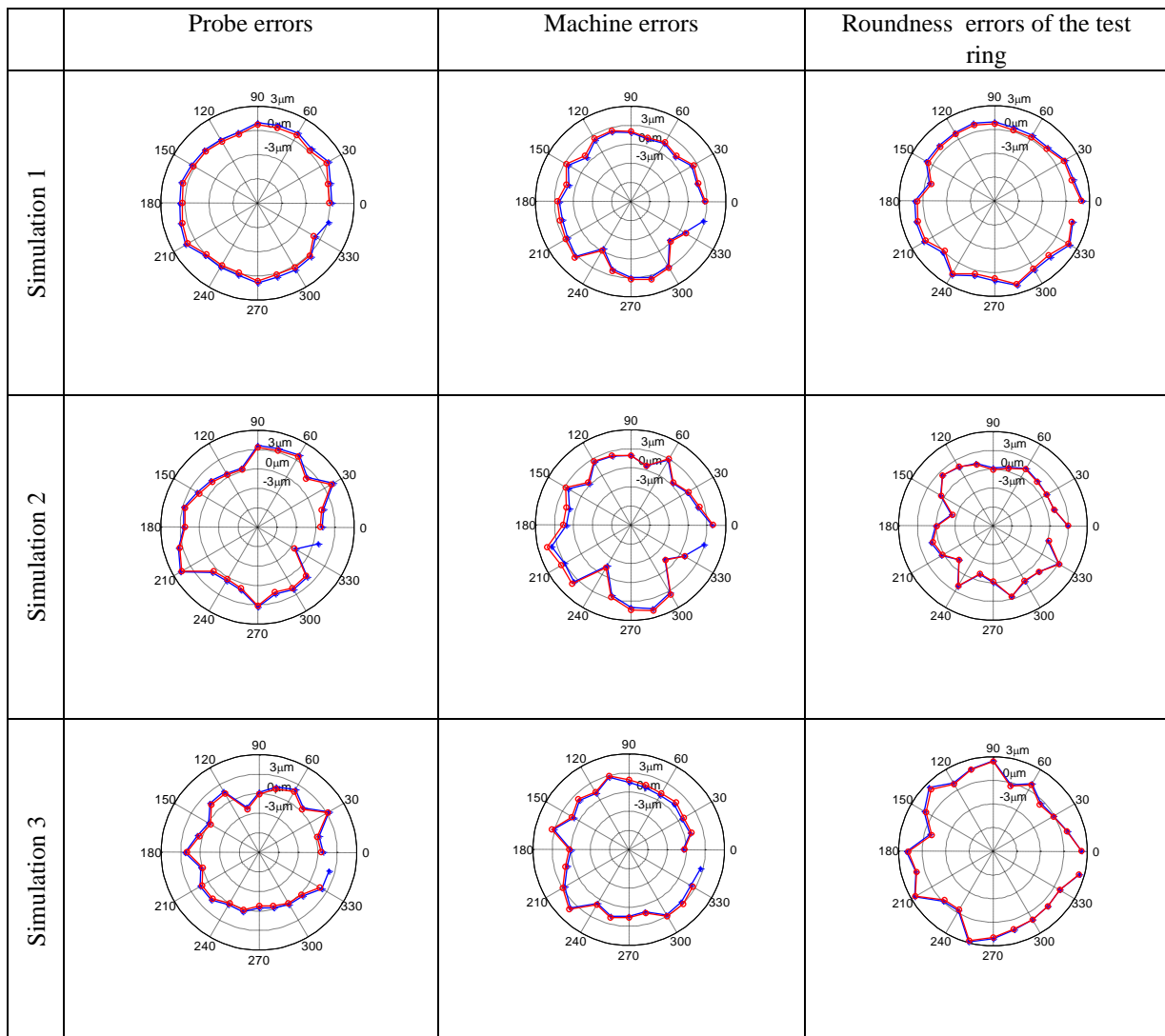
roundness error). If the test ring is perfect, we can set one probe error or one machine error to evaluate probe errors and machine errors in all others measurement directions.

**VALIDATION OF THE DEVELOPED APPROACH**

In order to implement and validate this approach, a virtual machine is created and several simulations are done. For each simulation, one probe error and one machine error are set arbitrary to zero.

We have, randomly, generated probe errors, machine errors and roundness errors of the test ring, the values of errors were taken within intervals representing real cases and each error is associated with an uncertainty. The test ring is probed at 24 points.

The combined errors, associated to the generated errors, which represent the measurement errors taken at the test ring, are used in the identification system to find machine errors, probe errors and out of roundness errors separately. Figure 4 presents an example for four simulations. It presents a comparison between the randomly generated errors and the identified errors using this approach, It shows that identified errors are in good agreement with those generated randomly, there is an offset between them, it depends on the two errors that were set to zero by removing the associated columns from the identification system. The method can be used to determine the probe errors variation, the machine error variation and the roundness errors of the test ring.



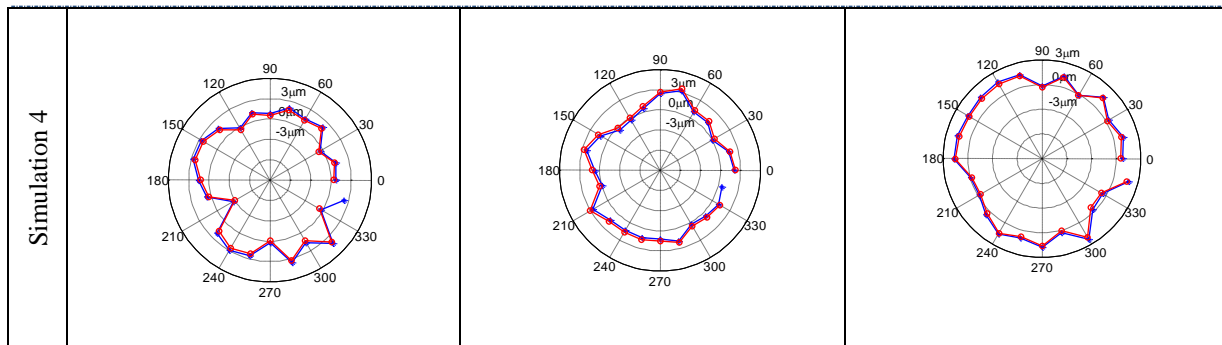


Figure 4: Comparison between randomly generated errors and separated errors for several simulations

—x— Generated errors      —○— Separated errors

To clarify the effect of the two set errors (probe error and machine error) on the variation of roundness errors, we performed several simulations by varying the set (probe error, machine error). For each configuration, we determine the difference between the maximum value and the minimum value of the simulated profile representing geometric shape of the test ring (Figure 5). It shows that whatever the set (probe error, machine error) imposed in the identification system, the variation of the identified roundness error could be realistic.

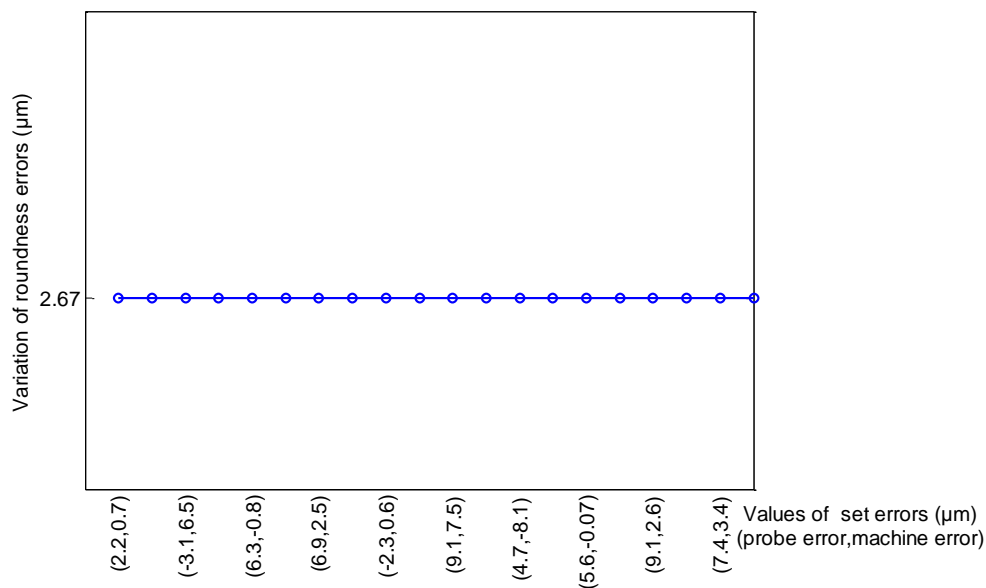


Figure 5: Effect of the two fixed errors on the variation of roundness errors

## CONCLUSION

A novel application and study of the separation method are presented to measure the roundness error separately to the machine errors and probe errors. The approach is based on redundancy measurements of the test ring. The identification system linking the measured radial residuals and the three sources of errors is deficient by two. To solve the system and have the absolute value, we have to set at least two errors which have not the same origin. Several virtual machines are tested and it proves that this approach is realistic.

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